Results in Engineering 27 (2025) 106276

Contents lists available at ScienceDirect

Results in Engineering

journal homepage: www.sciencedirect.com/journal/results-in-engineering

Research paper

Method for establishing reference limit values for vibration measurements in machines

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ARTICLE INFO

Keywords:
Vibration Measurement
ISO 20816
Vibration Reference Values
Machine Functional Condition
Predictive Maintenance

ABSTRACT

When measuring, the accuracy of the measurement is as important as the reference level used. There is little point in taking very accurate measurements if the reference levels used to evaluate them are inappropriate. Therefore, when measuring vibrations in rotating equipment, the general rule is to use reference standards, such as ISO 20816, or set alarm levels according to the manufacturer's recommendations. However, although setting specific vibration reference levels for each machine is recommended by international standards, there is no proposed method, or mathematical support, for doing so in either the standards or the related literature. This situation leads, in practice, to the use of general and, therefore, non-specific boundaries to assess the functional status of each particular machine (good, alert, alarm, etc.) and, consequently, to many false positives or, even worse, false negatives when detecting machine failures. The purpose of this study, and a novelty in this field, is precisely to present a method that allows statistically determining the most appropriate vibration reference levels for each rotating machine. Using a vibration database with measurements taken over twelve years for hundreds of machines, this study demonstrates that the vibration measurements of a rotating machine can be statistically modelled, with an accuracy close to 100 %, using the Rayleigh Distribution Function, which allows successive vibration reference levels to be formulated between functional categories from the mode and standard deviation of said statistical distribution function. The final section presents an example of the implementation of this methodology in an industrial plant.

1. Introduction

The intense globalisation experienced during the last decades of the 20th century and the beginning of the 21st century has offered companies great opportunities for growth. However, to do so, it is necessary to obtain the maximum and most effective use of existing facilities, optimise their operating conditions, and increase as much as possible the efficiency and performance of the equipment already installed, as well as its useful life. In other words, the current situation demands a commitment from companies to implement and develop maintenance strategies, especially predictive or condition-based maintenance, within a specific plan that includes a continuous improvement process focused on achieving the highest levels of performance [1]. Continuous measurement of significant functional parameters of equipment, particularly vibrations, is the core around which these strategies should be based. Having the correct reference vibration levels for machines is crucial here. If they are too high, silent failures can occur, but if they are too low, they can trigger constant alarms. Recurring false alarms from equipment go beyond simple operational issues. They can cause interruptions that affect productivity, but also confidence in daily work scheduling.

Since the end of the Second World War, vibration level measurement constitutes the cornerstone of predictive maintenance in dynamic equipment, mainly in rotating machines. Although there were numerous books on vibration theory, one of the earliest references that addressed diagnostic applications in rotating machinery, such as rotor balancing, critical speeds or propeller vibrations, was written in 1934: the pioneering and well-known book "Mechanical Vibrations" by J.P. Den Hartog, which has had numerous reissues to this day [2,3]. Even earlier,

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https://doi.org/10.1016/j.rineng.2025.106276
Received 18 June 2025; Received in revised form 11 July 2025; Accepted 11 July 2025
Available online 12 July 2025

Nomenclature

b	Basis of an exponential function
B_L	Basal Reference Level
c	Coefficient of an exponential function
c_i	Criticality criterion i
$C_{p,i}$	Parametric category i of parameter p
E	Exponent of an exponential function
$f(x)$	Real load probability density function
$F(x)$	Cumulative real load probability density function
$f(x)_{Theoretic}$	Theoretical load probability density function
$F(x)_{Theoretic}$	Theoretical cumulative load distribution function
$g(x)$	Real resistance probability density function
$g(x)_{Theoretic}$	Theoretical resistance probability density function
K	Number of intervals or classes in a set of measurements
KS_{value}	Maximum value of the difference between $F(x)_{Theoretic}$ and F(x)
L_i	Criticality level i
L_p	Number of parametric categories of the parameter p
$l_{p,i}$	Limit level i of parameter p
$l_{p,i}{}^*$	Corrected limit level i of parameter p
$l_{p,min}{}^*$	Corrected minimum limit level of the parameter p
m	Mode of the theoretical probability distribution function
s	Sample size of a set of measurements
n	Number of limit levels
r	Acceptable rejection risk level in the Kolmogorov-Smirnov Test
$V_{cr}i_{tic}al$	Critical value for the Kolmogorov-Smirnov Test
$\bar{x}$	Mean value of a set of measurements of variable x
μ	Mean value of the theoretical probability distribution function
σ	Standard deviation of the theoretical probability distribution function

in 1928, the book "Vibration Problems in Engineering" by S. Timoshenko and D.H. Young already included some sections applied to machinery diagnostics, which has also been reissued several times subsequently [4, 5].

It was around the 1970s and 1980s that the first technical books and articles dedicated exclusively to machine diagnosis using vibrations began to appear, primarily through spectral analysis, but also through waveform or shaft orbit analysis. It is worth mentioning as a first milestone in this field the so-called "Shore Table" (1968), which contains the spectral characteristics of a large number of common defects in turbomachinery [6,7]. One of the first books completely focused on technical personnel dedicated to the diagnosis of rotating machinery was "A practical vibration primer" by Jackson, C. (1979), which uses ISO 2372 and ANSI/API 610 standards as references to establish the condition of the machine [8]. At this point, it is worth noting that the aforementioned API 610 standard takes the vibration limit values given by the ISO 13,709:2003 standard, for centrifugal pumps for the petroleum, petrochemical and natural gas industries (updated in 2009). Domadiya [9] has tabulated some differences between different standards in the maximum vibration level recommended in centrifugal pumps. Since then, numerous books have been written entirely dedicated to the diagnosis of rotating machinery, some of which have had a great impact on industrial practice, such as John M. Vance's successful books "Vibratory machinery: Measurement and analysis" [10] or its more recent and improved edition "Rotordynamics of turbomachinery" [11], in which the diagnosis based on spectral and modal analysis is deepened. They also deserve to be highlighted in the field of rotating machinery diagnostics the books of Scheffer and Girdhar (2004). "Practical machinery vibration analysis and predictive maintenance" [12], focused on the detection and diagnosis of failures in rotating and reciprocating machinery using vibration signal analysis; "Machinery failure analysis and troubleshooting: practical machinery management for process plants", by Bloch and Geitner (2012), a common reference in industry that presents real cases of failures in rotating equipment detectable through vibrations [13]; or "Rotating machinery vibration: from analysis to troubleshooting" by Adams (2009), which addresses vibration analysis in rotating machinery, but in this case also uses computational signal modelling [14].

There are currently a large number of studies, books, and articles on this topic. New techniques and methods for detecting and diagnosing faults in rotating machinery, primarily through vibration signal processing, are proposed, as are interesting reviews. Worth mentioning is the review by Romanssini et al. [15], which provides a widespread analysis of vibration monitoring techniques for rotating machines. Although this article indirectly mentions the importance of establishing vibration limits, does not specifically focus on detailing maximum permissible levels or on the classification of functional categories. Also worth mentioning is the review by Tiboni et al. [16], which presents an interesting table with the main publications on condition monitoring in rotating machinery. Furthermore, the authors identify the main phases of the diagnostic process and propose the most relevant techniques used in each of them, focusing on the analysis of vibration signal patterns. While the work mentions that detecting vibration level anomalies is the first phase in the diagnostic process, it does not provide threshold values or specific references on acceptable vibration limits.

In recent years, the potential of artificial intelligence methods has attracted significant attention in this field, as in all disciplines. This is the case of the study by José López [17], who lists and briefly reviews advanced methodologies for rotating machinery diagnostics, including the use of artificial intelligence for automatic diagnostics. Similarly, Daga and Garibaldi [18] also review modern vibration signal processing techniques (FFT, STFT, Wavelets), pattern analysis and machine learning for diagnostics, including applications with neural networks, but they focused on the diagnostic process rather than the vibration levels that trigger these processes, as do Matania et al. [19], who offer a review of recent literature on deep learning applied to vibration-based fault diagnosis. Murtaza et al. [20] also review the use of new computational technologies (such as machine learning) to detect vibration anomalies or faulty operations, but by analysing the vibration signal. Similarly, and reinforcing what is discussed later about the widespread use of standards, Ugechi et al. [21] developed a computer-based diagnostic tool that uses an artificial neural network to diagnose machinery faults by analysing the vibration signal, but using the values given by the ISO 12,372 standard as reference vibration levels.

As seen above, the studies found in the literature on condition-based maintenance, vibration monitoring, or machine diagnostics focus on very specific aspects, whether measurement, signal acquisition or processing techniques, or diagnostic techniques, etc., but they generally use international standards such as ISO 20,816, API 610, or others as a reference for vibration limit levels, without any discussion on the matter. This is the case of Jin et al. [22] that question the use of fixed reference limits to assess machine condition. They propose a probabilistic method, based on feature density, for the automatic real-time detection of abnormal vibrations during the machine degradation process. Although, ultimately, they use the threshold values of ISO 10,816 as a reference to classify machine health zones. In this regard, Delprete and Gastaldi [23], in a very recent article (2025), state that - literally- their "paper constitutes, to the authors knowledge, the only systematic study in which the recommendations of all available international standards for vibration monitoring of non-rotating parts are applied to the same test cases and compared". In the study, they analyse two cases to compare the differences between several international standards and conclude that there is no general rule for establishing vibration limit values.

At this point, it is worth highlighting that, ultimately, the origin of vibration limits, whether from standards or manufacturers, were established - literally from ISO 2372 - "on the basis of theoretical considerations and practical experience" [24], in short, as Nelson states [25], based on empirical criteria, industrial experience, and field tests. Moreover, ISO 20,816–3 standard, states clearly that “the numerical values presented are intended to serve as guidelines based on worldwide machine experience, but shall be applied with due regard to specific machine features which can cause these values to be inappropriate” [26]. Interestingly, the limits set by ISO 2372:1074 [27] were exactly the same as those of its predecessor, the German standard VDI-2056, dating from 1964 [28]. Moreover, subsequent updates of ISO standards for rotating machinery have kept the same values, although they offer more general recommendations, in particular ISO 10,816–1:1995 [29] and ISO 20,816–1:2016 [30], which introduces more detailed criteria by machine type, operating environment and criticality. This latest update to the standard clearly highlights the importance of establishing machine-specific vibration reference levels beyond those established by the standard, however, it does not develop or propose any method for doing so. For this purpose, ISO/TR 19,201:2013, revised and confirmed in 2024 [31], provides guidance for selecting appropriate vibration standards, but ultimately points to the fixed values given by other ISO standards.

The aim and novelty of this work in the field of predictive maintenance is that it provides a methodology that allows, firstly, to statistically characterise the vibration of a rotating machine, more specifically, the representative value of the "normal behaviour" of the vibration measurement on non-rotating parts of each machine analysed. Second, and based on these basal reference values, to establish different boundary levels to classify its functional condition, that is, specific alert indicators regarding the machine’s status.

2. Vibration severity measurement

The usual procedure is to measure and evaluate the vibration signal at different points of the machine, whose value and variation are closely related to its functional status. If any of these measurements exceed certain limits (threshold or boundary levels), the causes will be analysed, normally by means of spectral analysis, so as to obtain a diagnosis that allows the necessary corrective actions to be taken to restore the equipment to its correct functional condition.

Maximising the chances of success in the implementation, adaptation, and/or modernization of predictive maintenance plans requires caution and appropriate expert guidance, especially when measuring machine vibration, in order to avoid common mistakes such as applying the same level of supervision to all machines, that is, not considering the different degrees of criticality of each machine; using the same parametric limits when evaluating machine conditions (correct/alert/alarm); or setting excessively restrictive or permissive limit values when we evaluating their functional status. It should be noted that predictive maintenance experts frequently modify the levels between states (alert, alarm, shutdown…) based on their own experience, that is, without following any systematic method beyond increasing the limit values, at its discretion, evidenced from the machine’s historical data, or specific manufacturer recommendations [32–36], to avoid constant alarms. The objective of this action is to adapt these levels to the "normal behaviour" of each machine, thus avoiding continuous false positives when the vibration trigger level is too low, or false negatives in the opposite case, that is, when the established reference level is too high. This action is contemplated by international standards, as stated above, with the objective of adapting these limits to the particularities of each machine, but they do not propose any methodology for this.

The different parametric categories to be established and their numbers depend on the standard or parametric references used for the machines. Between each functional category (good, alert, etc.), there is a value or reference level that separates one category from the adjacent one. Therefore, the number of boundary levels is equal to the number of categories minus one. For example, if we consider the simplest case with only two categories (good and failure), then there is only one boundary level that separates one state from the other. Although the limit level is a numerical value that does not generate doubts, the exact meaning of each category depends on the criteria used, that is, the extent to which a machine can be considered to be in *good condition, in alert condition, in alarm condition, etc.*, and must be specified for each case independently. This means that it is very important that the limit levels are chosen appropriately because these conditions categorise the functional status of the machine and, consequently, the actions to be carried out.

Regarding the importance of correct categorisation, international standards such as *Standard ISA-TR18.2.5–2012: Alarm System Monitoring, Assessment, and Auditing* [37], as well as many authors [38–43] also stated that if this categorisation process is carried out properly, undesirable situations such as stoppages of machinery or unnecessary interventions (commonly referred to as false positives) can be avoided. Hereafter, using the nomenclature introduced by Gómez de León et al. [44], for each parameter p subject to evaluation (vibration, temperature, etc.), each of the limit values between the functional categories of the machine (good, alert, alarm, etc.) set in the corresponding standard is denoted as $l_{p,i}$, where i is the level of the index, that is, a value indicative of its importance within the scale. Generally, a higher index value indicates worse machine conditions.

Following the same identification criterion, each of the above limit levels is designated in this work as $l_{p,i}*$ after having been corrected, that is, after having been adapted to the particularities of each machine. These types of modifications to the parametric levels are recommended by international standards themselves, such as the vibration standard ISO-20,816 [30], which states that, in certain cases, there may be specific circumstances and/or characteristics associated with each machine and/or equipment that require an adjustment of the limit levels to ensure that the requirement is realistic.

Although such modifications are recommended, establishing the new corrected levels $l_{p,i}*$ requires, on the one hand, proven knowledge of the dynamics of the particular machine and, on the other hand, extreme caution with the changes to avoid dangerous errors with the new values adopted, regardless of whether the modification is upward or downward. In the first case, that is, when the levels are modified upward and $l_{p,i}* > l_{p,i}$, the danger lies in promoting a *false negative* scenario, that is, a situation in which a defect or problem present in the machine can be ignored. In other words, the danger lies in placing the state of the machine in a functional category of “correct” or good, when in fact it’s not. On the other hand, when modifications are made downward and $l_{p,i}* < l_{p,i}$, the danger lies in the promotion of a scenario completely opposite to the previous one, which can be called a *false positives* scenario, in which the state of the machine is qualified as incorrect when it is not really incorrect. This circumstance causes undesirable situations, such as unnecessary stoppages and/or unnecessary actions, with consequent economic expense and loss of time, as well as the added risk of causing some damage during the intervention.

Although these modifications are common in practice, the reality is that today, there is no standard methodology for this task, which means that the responsibility and degree of success lie solely with the technical personnel who perform them. This fact also entails a number of disadvantages, such as subjectivity, the presence of external influences, the existence of preconceived ideas, and the lack of homogeneity in adjustments.

All vibration standards, including those from normalization organizations, manufacturer recommendations and engineering rules of thumb for plant maintenance, are derived from empirical observations. From the Rathbone Chart (in the 1930s) [45] up to the most recent standards, such as ISO 20,816–1:2016 (revised and confirmed in 2021), a mathematical expression has been sought to standardize the sequence of alarm levels established through experience with plant equipment. In that sense, from the beginning of the 20th century to the present, exponential distributions have played a crucial role in modern engineering reliability. They are widely used to predict failure rates, as a statistical model to estimate the probability of a component failing randomly. Although alarm levels are a very short sequence of experimental values, the use of an exponential function to model them has been the norm from the beginning. Nevertheless, there are some exceptions, mainly from instrument or equipment manufacturers, such as the use of linear (Alfa-Laval decanter centrifuge [35] or Adash Temperature Limit Values [32]) or polynomial (Adash Vibration Limit Values [32]) alarm scale. Furthermore, by using exponential functions, a logarithmic scale can be drawn to represent the values linearly, which considerably simplifies predictions, especially before digitization, i.e. when working with paper.

In order to provide practical flexibility in setting alarm levels, beyond a mathematical formulation model, some predictive maintenance programs [46] offer the possibility of using the mean value of the last 5, 10, 15, … measurements as a reference value when setting alert and alarm values. However, this strategy can be dangerous, because if the measurements are not properly filtered and evaluated, these mean values evolve in such a way that they hide the real problem in the machine. Therefore, apart from directly applying the values given by the standards, neither these international vibration standards nor the specific literature provide a method for mathematically adapting and setting the boundary values between functional categories for a particular machine, i.e., good, alert, or alarm condition.

The random nature and consequent degree of diagnostic uncertainty involved in the evaluation of machine vibrations, which is a feature of the field of reliability engineering, suggests that, assuming all its advantages and limitations, the best method to find a solution to this problem is to opt for a probabilistic approach, as was done in this work, and more specifically, to study the functional state of the machine as a load-resistance analysis.

3. Load-Resistance analysis

3.1. Fundamentals

Within the branch of reliability engineering, defined according to Yáñez et al. [47], as “the branch of engineering that studies the physical and random characteristics of the failure phenomenon”, two different schools can be distinguished. The first and oldest are based on a probabilistic analysis of the time between failures, and the second and currently more widespread is based on a probabilistic analysis of the failure characteristics themselves.

Despite the good results achieved throughout history with studies based on the first approach, during the last few decades, the implementation of methods based on the study of the characteristics of the physics of failure has increased significantly. This is mainly owing to the increasing possibility and ease of acquiring and storing data, as well as a greater ability to analyse interrelationships between multiple parameters that indicate the physical and functional state of machines. This type of failure physics analysis is based on the so-called load-resistance analysis, the fundamental premise of which is that failures are the result of situations in which the load supported by the machine is greater than its mechanical resistance at a given moment [48].

Assuming that the measured parameter, vibration in this case, reflects the functional conditions of the machine within the scope of the application of this study, the concepts of load and resistance should be understood as the point values present at a given instant and the limit values that have been established for these physical quantities, respectively. That is, the load is represented by the values of the vibration existing at the time of measurement, which must be interpreted as the functional conditions to which the machine is subjected at that moment, whereas the resistance must be associated with the limiting severity levels (alert, alarm, etc.) that the machine can withstand before failure.

To clarify this concept, consider the example of an industrial reactor with a maximum design pressure of 10 bar, which is subjected to real process conditions of 4 bar at a given instant. In a hypothetical load-resistance analysis, it was observed that the load conditions were below the maximum resistance. Therefore, it can be stated that the equipment was working under reliable conditions. On the contrary, if, in an instant, the pressure reaches a value of 11 bar, that is, above the limit value, although it cannot be assured that failure will occur with certainty, it is clear that the probability of failure will be more than considerable.

Although in real situations, similar to the one described above, concrete or deterministic values are used for both load resistance values, the empirical reality is that these values represent statistical properties and, therefore, are statistically distributed according to the probability density functions, as shown in Fig. 1.

The situation represented in Fig. 1 corresponds to a hypothetical initial situation in which the machine is in a perfect functional condition because at no time can the load acquire severity levels that exceed the lowest of the probable values of its resistance. This functional safety situation recedes as the machine’s life cycle progresses as it undergoes progressive degradation caused by both its own operation and the action of external agents, which eventually reduces its resistance and causes its failure.

Similarly, even if the machine retains its initial strength, the load may increase if the operating conditions are inadequate, such as when the machine has a misalignment between the shafts, an unforeseen overload, or a rotor imbalance, among other well-known functional defects. A decrease in the machine strength implies a migration, or at least an extension, of the strength probability density function *g(x)* to the left of the graph; that is, the machine would withstand lower levels of stress. In the presence of a defect, this migration is logical, because a defect in a machine is, by definition, a decrease in certain resistance capabilities. However, the presence of any defect causes the machine to suffer higher stresses which implies that the load curve *f(x)* shifts or at least extends to the right of the graph, that is, towards higher values.

3.2. Approach to set alert level

Because of both migration and extension, functions *f(x)* and *g(x)* approach each other and may even overlap, as shown in Fig. 2. This overlapping means that there is a certain probability that the load supported by the machine exceeds the possible resistance values; that is, the probability of failure is greater than zero. Logically, the larger the intersection zone, the greater is the probability of failure.

Therefore, it seems reasonable that the value at which the overlap of both curves begins marks the first alert level. Therefore, the lowest corrected limit levels, $l_{p,\min}$*, were set, as shown in Fig. 3. Thus, it can be assumed that, below this level, the machine works safely, whereas with values above this level, there is a certain probability of failure. Consequently, it can be considered an alert level.

This statistical argument is the basis of the proposed methodology, so that the first alert level $l_{p,\min}$* is established, and successive higher levels

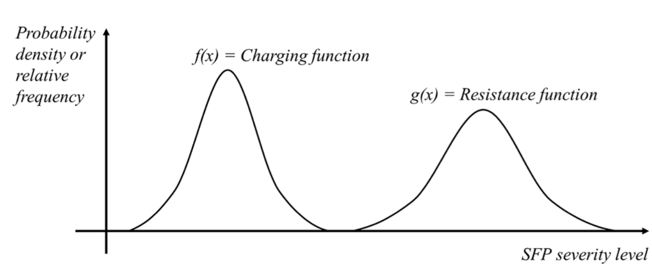

Fig. 1. Probability density distributions of load and resistance.

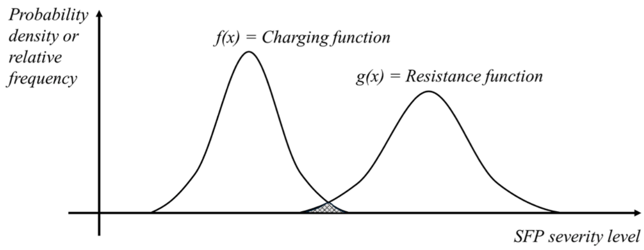

Fig. 2. Overlap between load and resistance probability density functions.

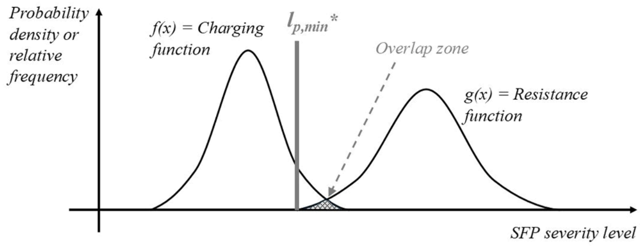

Fig. 3. Location of the minimum limit level $l_{p,\min}$* (alert level) in the overlap zone.

(alert, alarm, stop, etc.) must be set according to the associated failure probability, that is, according to the overlapping area between the two curves or probability density functions.

As in any statistical analysis, the basis for establishing these levels is the historical record of data, in this case vibration measurements.; therefore, the reliability of the available dataset will condition the degree of adjustment of the statistical models of both curves to reality, and consequently, the goodness of the calculated limit levels. Therefore, the values that make up the dataset *f(x)* are those measurements in which it has been verified that the machine in question is in the correct functional condition, whereas the data that make up the resistance function *g(x)* must be the contrasted results of some defects present in the machine, such as misalignment, unbalance, cavitation, and deteriorated bearings.

Because there are specific cases in which there is an insufficient volume of data to adequately model functions *f(x)* and *g(x)* (for example, the case of a new machine), the proposed methodology has a stage called *generic modelling*, which focuses on identifying repetitive patterns in machines with similar characteristics to the factory itself and can serve as a guide when determining the most appropriate particular levels of $l_{p,i}$*. This is what classically happens with general standards, such as ISO 20,816 or any others, but it is also the direction in which novel artificial intelligence techniques, such as Deep Learning, referred to by Matania et al. [19] as Transfer Across Different Machines, are heading.

This modelling stage consists of the determination of the theoretical function $f(x)_{Theoretic}$ that best models the vibration values representing the real load *f(x)* of the machine, as well as the calculation of its statistical properties (mean, mode, standard deviation, etc.) to determine their formal relationship with the experimentally obtained $l_{p,\min}$* level.

In summary, this stage aims to find a reliable statistical model, $f(x)_{Theoretic}$, capable of representing the real load measurements of a set of similar machines, and to check whether the characteristic values of this function can be related by a formal expression to the experimentally obtained $l_{p,\min}$* levels.

4. Methodology for determination of corrected limit levels

This methodology is intended to provide a tool for determining the corrected vibration limit levels $l_{p,i}$* for each machine in an objective and systematic manner. For this purpose, it uses as basis the load probability density functions *f(x)* and resistance *g(x)* generated from the historical record of real measurements of the machine, where the variable *x* is the measured vibration level.

This empirical orientation of the work is essential in any statistical analysis and is in line with the approach of Sastre [48], who indicated that any study within the field of application of reliability engineering must combine mathematical techniques or tools with an intensive use of the existing historical data.

Fig. 4 presents a flowchart summarising the stages of this methodology, which is explained in detail in the following subsections.

4.1. Determination of the real probability density functions of the machine vibration

The first stage of the methodology begins with the collection of data and their subsequent classification with the intention of being able to represent their loading probability density function *f(x)*. This requires a sufficiently large volume of data to produce a reliable histogram of relative frequencies. To elaborate on the histogram, it is necessary to begin by ordering the set of measurements according to their severity and then proceed to the calculation of the range of the set and the classification of the measurements into different intervals or severity classes. This last point is essential because the vibration is a continuous variable that can acquire any specific value in the set of real numbers, the only restriction being the precision of measurement. The proper distribution of data intervals, that is, determining their number and width, is essential for discovering the true distribution of the data.

There are different methods to perform this classification of data ranging from the simple *Velleman's Rule* [49,50] which consists of a simple square root of the sample size, to other much more complex and with more solid mathematical support such as the *Sturges Rule* [49,50], which is valid for normal distributions and small sample sizes, or the *Dixon-Kronmal Rule* [49,50], the *Freedman-Diaconis Rule* [49,51,52], and *Rice's Rule* [49,51,52] which are recommended for cases in which the sample size is larger.

In this study, where the historical data of the measurements of each of the machines are hardly going to be counted in thousands, the *Rice Rule* has proven to be the most appropriate, showing results most consistent with reality. Although other methods, such as *Sturges Rule* also provide good results when there are few sample values ($n < 200$), for large values, it assumes that the distribution is Gaussian, as with *Scott's Rule*, which cannot be stated a priori in this case. On the other hand, the *Dixon-Kronmal Rule* behaves well in cases where the sample

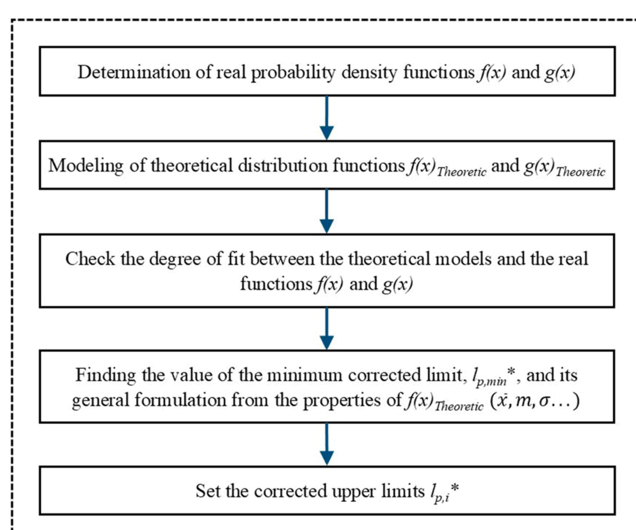

Fig. 4. Flowchart for the determination of corrected limit levels $l_{p,i}$*.

size is large; however, with a few hundred data or less it is no longer useful because it tends to fix too many intervals which makes it difficult to identify the trend of the set. Finally, the *Freedman-Diaconis Rule* also provides good results with the data; however, it considers dispersion in the interquartile range and is therefore less sensitive than *Rice Rule* to extreme values.

Fig. 5 shows a histogram of the relative frequencies corresponding to one of the machinery units analysed. The unit consisted of a centrifugal pump coupled to an electric motor.

After obtaining and representing the load probability density function *f(x)*, we proceed in the same manner to obtain the resistance probability density function *g(x)*. In this case, the main difference lies in the fact that the dataset used here consists only of the vibration severity measurements that are known to correspond to equipment in failure mode, that is, measurements that indicate the existence of a defect in the machine and therefore representing an incorrect functional condition of the machine. It should be noted that in the case of the function *g(x)* the dataset will obviously be smaller than that in the load function *f(x)* because it is normal for machines to operate in good condition, and their operation in failure mode is infrequent.

Fig. 6 shows the resistance probability density function *g(x)* corresponding to the same machine used as an example in Fig. 5 to represent the load probability density function *f(x)*. The joint representation of both functions allows a clear observation of the value of level $l_{p,\min}$*, which is the reference value used to establish the other limit levels between functional categories.

4.2. Modelling of distribution functions

The objective of the second stage of the methodology is to determine which of the theoretical probability density functions ($f(x)_{Theoretic}$ y $g(x)_{Theoretic}$) best model the distribution of the real values of the machine vibration under study. The aim is to find a theoretical model that follows the distribution of the data of each machine and a possible pattern to be used in similar cases where the volume of available data is scarce.

The simple observation of the shape of the distributions of the real data allows us to discard some candidate functions for modelling the data from the outset. In the case of parameterizable functions, such as the Gamma or Weibull Distributions [53,54], it is necessary to iterate the calculations using their parameters as variables until the values that best approximate the theoretical functions to the real ones are found; that is, it is necessary to check the degree of fit between the real distribution and the model functions by means of a mathematical procedure that supports the validity of the modelling.

Fig. 7 shows the distribution of the real data previously shown, together with the three possible theoretical models: the Rayleigh distribution function (a particular case of the Weibull function in which the shape parameter takes a numerical value 2) [55], normal distribution function, and log-normal distribution function.

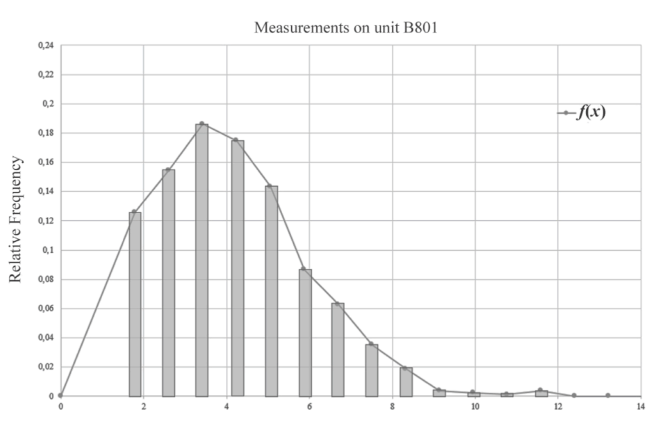

Fig. 5. Histogram of relative frequencies for vibration level measurements on a centrifugal pump with intervals calculated using the *Rice Rule.*

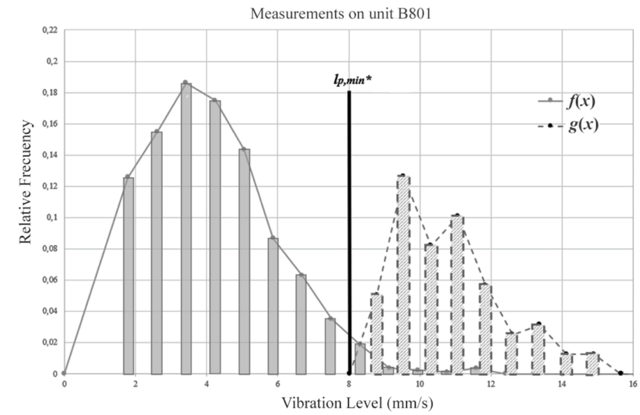

Fig. 6. Overlap of the load, $f(x)$, and resistance, $g(x)$, histograms to locate the minimum corrected limit level $l_{p,\min}$* (alert level) of a centrifugal pump.

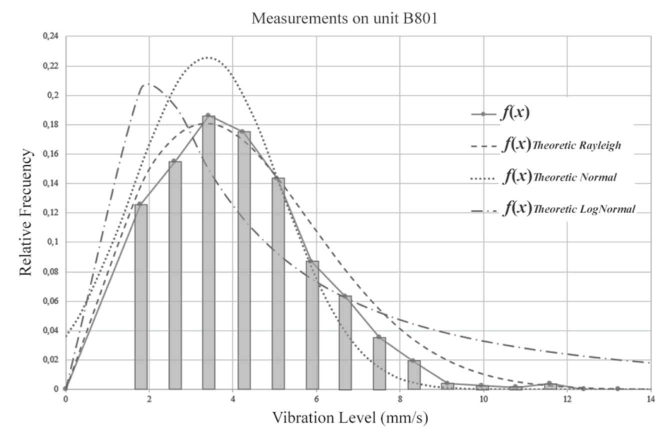

Fig. 7. Representation of theoretical distribution functions on an empirical histogram of vibration level measurements on the *B801* centrifugal pump.

4.3. Testing the goodness of fit between the theoretical and real functions

Although the graphical representations visually illustrate the functions that can best model the real load and resistance distribution functions, the process must be completed with a quantitative verification of the goodness of fit, i.e., “the probability a theoretical distribution has of reproducing the data set of a real sample” [48]. This verification is carried out by means of the so-called *Goodness of Fit Tests*, among which we can cite the *Lilliefors Test* [56–58] or the *Shapiro-Wilk Test* [57,58], applicable when one wants to check whether a data set conforms to a normal distribution; the *Chi-Square Test* [50,52], widely used but with weaknesses associated with the determination of the number of intervals; the *Kolmogorov-Smirnov Test* [56–59], based on the comparison between the theoretical and real cumulative probability functions, or the nonparametric *Anderson-Darling Test* [59], which, although similar to the previous one, has greater sensitivity at the extremes of the distribution.

For the reasons given above, the most recommended goodness-of-fit test in this case is the *Kolmogorov-Smirnov Test*, because although the *Anderson-Darling Test* is more sensitive to extreme values and this is something that must be taken into account in the case of the vibration measurements, these values are not really representative of a normal function but mostly correspond to anomalous punctual situations that are unrelated to the functional state of the machine itself, such as punctual overloads, start-ups of other nearby machines, and the influence of interconnected elements. The *Kolmogorov-Smirnov Test* is based on a comparison between the real and theoretical cumulative distribution functions, $F(x)$ and $F(x)_{Theoretic}$, as shown in Fig. 8.

From a mathematical perspective, this type of goodness-of-fit test is conducted by comparing two values: KS_{value} and $V_{critical}$. To consider the fit hypothesis valid, that is, the theoretical function adequately models the real function, the following condition must be fulfilled:

$$KS_{value} < V_{critical} \tag{1}$$

To calculate the second of these values ($V_{critical}$) the sample size *"s"* and the desired significance level must be considered. These values are respectively the total number of vibration measurements available and the risk *"r"* to be assumed with respect to the rejection of the established adjustment hypothesis. Table 1 shows the values that this critical value can acquire, depending on these two variables.

On the other hand, the KS_{value} value takes the value of the maximum of the difference between the theoretical and the real cumulative distribution functions, i.e.:

$$KS_{value} = max(|F(x_i) - F(x_i)_{theoric}|) \tag{2}$$

where:

$F(x_i)$: Cumulative distribution function of the data at point *i*.

$F(x_i)_{Theoretic}$: Theoretical cumulative distribution function at the same point *i*.

Although there may be different cases in other types of machinery, after analysing several hundred pieces of rotating dynamic equipment (including different types of pumps, compressors, electric motors, gearboxes, fans, agitators and mixers, belonging to a variety of industrial facilities, such as chemical plants, an oil refinery, several wastewater treatment plants or food industry plants), with thousands of vibration measurements, the results of this work agree with those of previous studies [44,60], in which it was established that the Rayleigh distribution function (3) is the best statistical model for representing the vibration data of a rotating machine. Moreover, it is a probability distribution that has been used by other authors to model mechanical vibration in other fields [61–64], particularly in the case of vibration values as a consequence of the fatigue or aging of materials [65,66]. The Rayleigh probability density function for the vibration variable (x) is given by Eq. (4), and its defining statistical property is the mode (m) of the distribution.

$$F(x) = 1 - e^{-\frac{x^2}{2m^2}} \tag{3}$$

$$f(x) = \frac{x}{m^2} e^{-\frac{x^2}{2m^2}} \tag{4}$$

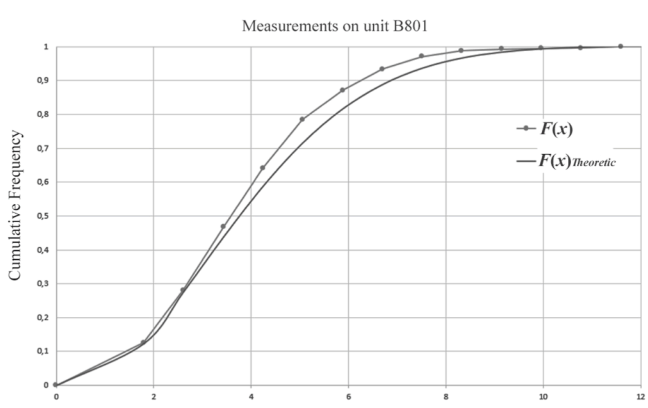

Fig. 8. Real and theoretical cumulative distribution functions ($F(x)$ and $F(x)_{Theoretic}$, respectively) of the *B801* centrifugal pump.

Table 1
Critical $V_{critical}$ values for the Kolmogorov-Smirnov test.

s	*R*				
	0,2	0,15	0,1	0,05	0,01
1	0.900	0.925	0.950	0.975	0.995
5	0.446	0.474	0.510	0.565	0.669
10	0.322	0.342	0.368	0.410	0.490
15	0.266	0.283	0.304	0.338	0.404
20	0.231	0.246	0.264	0.294	0.356
25	0.210	0.220	0.240	0.270	0.320
30	0.190	0.200	0.220	0.0240	0.290
35	0.180	0.190	0.210	0.230	0.270
>35	$1.07/\sqrt{n}$	$1.14/\sqrt{n}$	$1.22/\sqrt{n}$	$1.36/\sqrt{n}$	$1.63/\sqrt{n}$

4.4. Determination of the minimum corrected limit level and its relation to the theoretical distribution function

As shown in Fig. 6, for each machine, the first alert level $l_{p,\min}{}^*$ corresponds to the starting point of the function *g(x)*. However, when the volume of data available to obtain and represent the above functions is insufficient, the probability density functions of the mode corresponding to a class of similar machines whose modelling has been obtained should be used as a reference.

As with international standards, having a valid model for a given set of machines of the same class provides a solid reference for contrasting the state of any similar machine, regardless of whether it has its own reference vibration level. This avoids possible biases that may occur gradually in the real load distribution function and, therefore, in its characteristic values due to aging or any other circumstance of gradual evolution. That is, since each machine will have its own statistical distribution, by considering a significant number of similar machines, a general characterization can be made that represents this group or class of machines, just as international standards operate. In this way, even when the average of the successive measurements of a machine tends to slowly evolve toward abnormal values, this deviation can be detected having the overall mean value as a reference.

Fig. 9 shows an example of a real case of modelling a class of analysed machines, as well as the location of the $l_{p,\min}{}^*$ level and some statistical properties of the $f(x)_{Theoretic}$ function.

4.5. Determination of boundary levels between functional categories

Finally, once the level $l_{p,\min}{}^*$ is known, it is possible to establish the upper corrected limit values $lp_{,i}{}^*$, that is, those separating the different

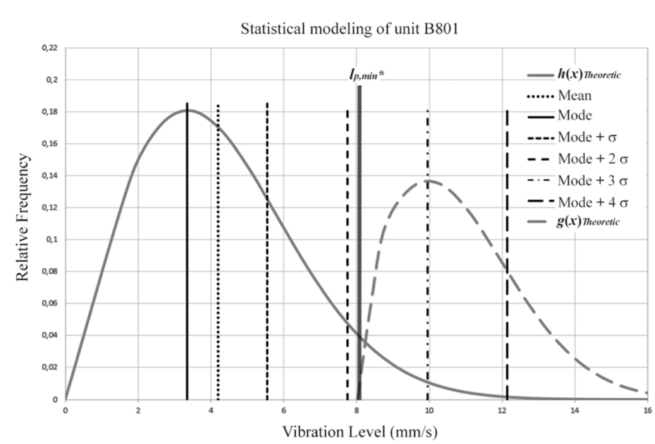

Fig. 9. Overlap of load and resistance theoretic distributions for a class of centrifugal pumps, showing the location of the alert level ($lp,\min^*$) and other significant statistical properties.

functional categories (Good Condition, Alert, Alarm, etc.), which we will designate following the criterion established by Gómez de León et al. [44], as categories $C_{p,i}$. To carry out this task, it is necessary to set both the number of boundary levels L_p-*1*, which will be one unit less than the number of defined functional categories L_p, and the mathematical relation that governs the jumps between these boundary levels appropriately.

At this point, it is important to remember that, regardless of the sequence of alerts or alarm levels that are set, we always deal with a space of probabilities. In other words, setting a certain level of alert for failure is equivalent to assuming a certain probability of success in failure diagnosis, but not its certainty, regardless of how high the probability is. When deciding how many alert/alarm levels should be set, one option is to resort to the same number of limit levels $lp_{,i}*$ set by some of the international standards related to the machine vibration under study (for example, ISO 20,816–1 sets three limit levels among its four functional categories: Good, Satisfactory, Unsatisfactory, Unacceptable) and uses the same mathematical relationship that governs the jumps or scaling between them.

In this study, different standards were analysed, and a mathematical expression that constitutes a generalisation of these standards was proposed. Thus, being parameterizable, it allows for better adjustment to each case (equipment, machine, and parameter) than general standards such as ISO 20,816–1 for vibrations.

The graphs in Fig. 10 display the sequence of values (every square point) that constitute the boundaries between different functional category levels, i.e., alarm severity levels, for several standards, namely, Rathbone Chart [45], IEC 60,034–14 [67], ISO 10,816–6 [68] and Trendline D_{eff} Bearing Levels [46], respectively. Each plot is accompanied by its trend line (dashed lines) using the R-squared regression analysis. As can be seen there is a perfect fit (R^2=1) with an exponential sequence of values.

With this approach, by analysing the sequence of limit values of most parametric standards, it is observed that they follow an exponential

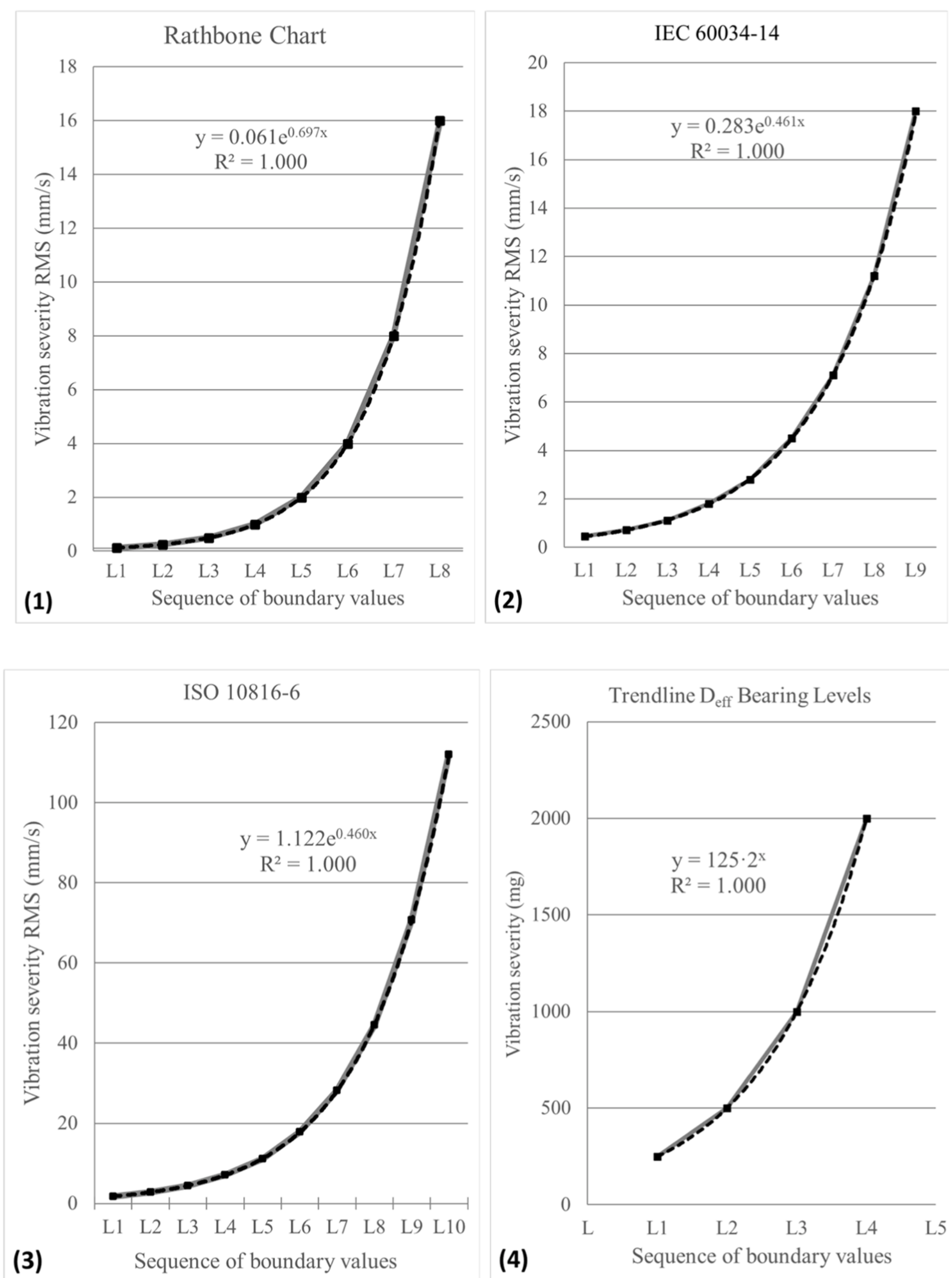

Fig. 10. Sequence of vibration severity alarm levels in Rathbone Chart (1), IEC 60,034–14 (2), ISO 10,816–6 (3) and Trendline D_{eff} Bearing Levels (4).

mathematical function to determine the limit values between functional categories, as stated in Section 1. Its mathematical expression corresponds to the following generalization (5):

$$l_i = B_L + c \cdot b^{E \cdot i} \tag{5}$$

where:

B_L: Basal reference level.

l_i: limit level between the parametric categories $C_{p,i}$ y $C_{p,i+1}$.

i: order of the limit.

c: coefficient of function (characteristic of the standard; $c > 0$).

b: basis of the exponential function (characteristic of the standard; $b > 0$).

E: exponent of the function (characteristic of the standard; $E > 0$).

In this way, for instance, in Fig. 10(3) B_L=0, $c = 1.122$, $b = e$, $E = 0.460$ and $x = i$, while in Fig. 10(4) B_L=0, $c = 125$, $b = 2$, $E = 1$ and $x = i$.

Because Eq. (5) is parameterizable, this expression gives rise to a wide range of possibilities. Because the starting reference level is $l_{p,\min}{}^*$, it is convenient to obtain the sequence of levels between functional categories starting from this level, $l_{p,\min}{}^*$, as shown in Eqs. (6) and (7) when B_L=0.

$$l_{i+1} = c \cdot b^{E \cdot (i+1)} = c \cdot b^{E \cdot i} \cdot b^E \tag{6}$$

so:

$$l_{i+1} = l_i \cdot b^E \tag{7}$$

Thus, taking the value $l_{p,\min}{}^*$ as the initial level for a standard with L_p functional categories, the calculation of the upper levels is performed as indicated in Eqs. (8) to (10), where n represents the number of limit levels, that is n=L_p-1.

$$l^*_{p,\min} = l^*_{p,1} = B_L + c \cdot b^E \tag{8}$$

$$l^*_{p,i} = B_L + c \cdot b^E \cdot b^{(i-1) \cdot E} \tag{9}$$

$$l^*_{p,n} = B_L + c \cdot b^E \cdot b^{(n-1) \cdot E} \tag{10}$$

In the specific case of vibration, its value, and therefore, its meaning when measured in units of speed, are closely related to the kinetic energy of the phenomenon. Therefore, the increase in the energy contained in the vibratory signal has a quadratic relationship with the increase in the vibration value.

As shown for the vibration case, the model conforms to the Rayleigh distribution, whose probability density function is given by Eq. (4) and its defining statistical property is the mode, m, of the distribution. As this is ultimately the most probable value when measuring using the machine, it acts as the basal reference level (B_L). Taking now the standard deviation, σ, as the statistical estimator to set the successive jumps between levels, the successive n limit levels to be set will be obtained from the following expressions, (11), (12) and (13):

$$l_{p,min}{}^* = l_{p,1}* = m + \sigma \cdot 2^{1 \cdot k} \tag{11}$$

$$l_{p,i}* = m + \sigma \cdot 2^{i \cdot k} \tag{12}$$

$$l_{p,n}* = m + \sigma \cdot 2^{n \cdot k} \tag{13}$$

where, observing the general expression (5), k corresponds to the exponent E of the function, the value of the base b takes the value 2 here, and the coefficient c is, in this case, σ, that is, the standard deviation of the distribution function. Thus, with $k = 1$, if we define, for example, the following four categories: *good, alert, alarm and stop*, the corresponding three limit levels ($n = 3$) would be: $m + 2\sigma$, $m + 4\sigma$ y $m + 8\sigma$, whose probabilities associated with a defect in the machine are 93 %, 99.8 % and practically 100 % (specifically 99.99999999990 %), respectively. This is a formal generalisation of Gómez de León's proposal [60].

Since the values of m and σ of the corresponding Rayleigh distribution are calculated individually for each machine from its historical measurements, from a practical point of view, it is much simpler to use an exponential function with a base of 2 instead of the number "e." This is not a problem because, since the number of alarm levels to be established is very limited -usually no more than 3 or 4-, the regression coefficient can be equal to 1 in both cases when adjusting the mathematical model, as shows Fig. 11 for the ISO 10,816–1 standard or the Trendline D_{eff} Bearing Alarm Levels.

It should be noted that the probability associated with a machine failure *alarm* level (which manifests as an atypically high level of vibration), given by $m + 4\sigma$ in the Rayleigh distribution, which is 99.85 %, coincides in practice (99.86 %) with the probability given by $m + 3\sigma$ in the Normal distribution, a level which is traditionally used to detect outlier cases in many fields, such as medicine [69], economy [70], climate [71], construction engineering [72,73] or to rule out outliers in vibration processing [74].

4.6. Practical application of the method in the case of vibration measurement

Having demonstrated that the Rayleigh distribution is the best approximation of the distribution of real measurements of machine vibration, given that the Rayleigh distribution has a single variable parameter, namely the mode (m) of the distribution (Eq. (3)), any other statistical properties, such as the mean μ and standard deviation σ of the distribution, can be obtained, since they are related to each other, as Eqs. (14) and (15) show.

$$m = \mu\sqrt{\frac{2}{\pi}} \tag{14}$$

$$\sigma = \mu\sqrt{\frac{4}{\pi} - 1} \tag{15}$$

From the perspective of its use in practice, once a sufficiently representative set of measurements of the vibration of the machine in good functional condition is available, it is sufficient to calculate the mean ($\overline{x}$) of that set of values to perform its modelling. Therefore, assuming that $\overline{x}$ is an acceptable estimate of the theoretical mean of the distribution μ by calculating the value of the mode m, through expression (14), and the standard deviation σ, using Eq. (15), the limit levels between the functional categories, given by Eqs. (11)-(13), will be defined.

In practice, 10–12 measurements is an initially representative number of measurements to estimate the mean (for a 90 % confidence level of the sample, i.e., $Z = 1.645$, with a margin of error of 0.71 mm/s - which is the lowest limit established by the ISO 20,816 standard for newly commissioned machines - and using the sample standard deviation as an estimator of the standard deviation of the distribution [75]). However, since as many measurements as desired can be made on the machine, the higher the number of samples (i.e. vibration measurements) the better the estimation of the model and, consequently, a higher degree of refinement will be achieved when establishing the boundaries that determine the functional condition of the machine (good, alert, alarm, etc.) such as: $m + 2\sigma$, $m + 4\sigma$, $m + 8\sigma$, etc.

4.7. Limitations and requirements of measurements

As mentioned above, the methodology described refers to vibration measured on non-rotating parts of rotating machines. Therefore, all measurements used in this work were performed using accelerometers, typically with a sensitivity of 100 mV/g, common in most vibrometers, fixed solidly near the machine bearings, as usual. Since the study and measurements spanned many years, different sensors (accelerometers), vibrometers, vibration signal collectors, and software were used, not

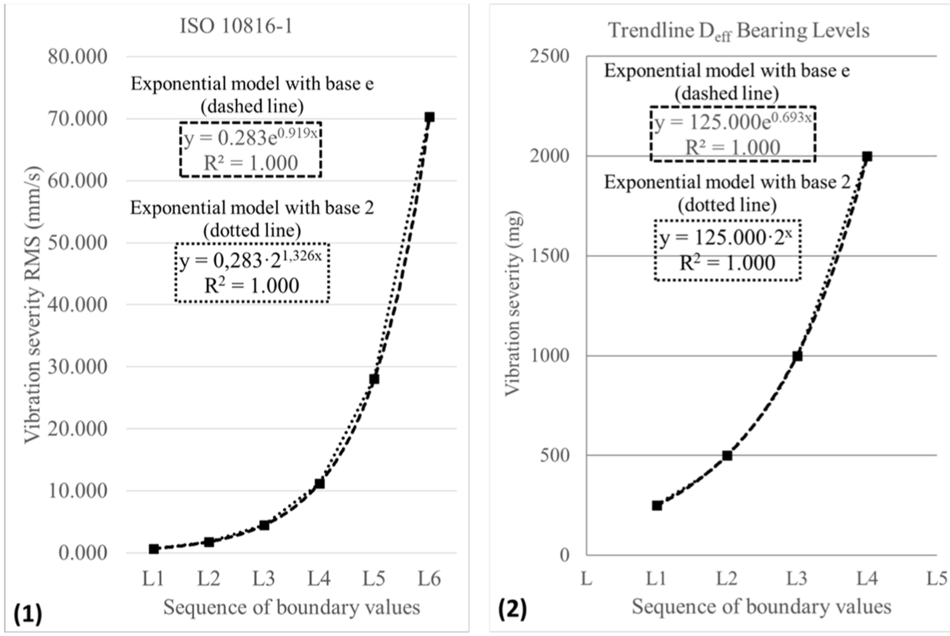

Fig. 11. Exponential modeling with base 2 and base e of ISO 10,816–1 standard (1) and Trendline D_{eff} Bearing Alarm Levels (2).

only over the years, but also to perform redundant measurements and ensure reproducibility of results. Although errors are inherent in any measurement process, the results obtained after thousands of measurements and their corresponding verifications are robust enough to support the proposed methodology and statistical model.

To precise the volume of data managed in this study: approximately 300 RMS vibration velocity (mm/s) measurements were taken per machine every year, over 100 production units, which represents at least two machines per unit, and for more than 10 years. To ensure results comparable with standards, the dynamic range used in the measurements was 10 to 1000 Hz for the RMS vibration velocity. However, other broader frequency ranges were also tested (particularly for low-speed rotating machines), as well as measurements in acceleration units (specifically to determine vibration level limits for bearing fault detection), which also confirmed the validity of the methodology even when measurements are based on a different dynamic range.

The main requirement for correctly applying this methodology to a machine is to accurately determine its basal vibration. This requirement is obvious, but it involves, first and foremost, taking as many measurements as possible while ensuring the machine is in good condition, i.e., ensuring its functional status is correct. It is important to measure covering all operating conditions under which the machine will normally operate to obtain a representative spectrum of vibrations that it will normally withstand, but always verifying that there are no faults present in the machine. If these requirements are met, as stated above, usually from 10–12 measurements are usually sufficient to make an initial estimate of basal vibration, specifically to calculate the mean value ($\overline{x}$) of these measurements, as a valid estimate of the mean of the distribution model (μ). Once the mean of the model is known, the boundaries for classifying the status of a machine after a measurement are immediate, as explained above. It should be noted that, following the methodology described above, the modelling of the function *g(x)* is unnecessary because the corrected limit levels obtained are already associated with machine failure probabilities, which is very significant from a practical point of view.

As with any statistical approach, the goodness of the model, and therefore of the alert and alarm levels, which are calculated to be used as a reference for the status of the machine, depend on the quantity and reliability of the measurements used to obtain the mean, mode, and standard deviation of the function *f(x)*.

5. Verification in a real case

5.1. Main data of the example

To demonstrate the validity of the model proposed using the method described above, the entire procedure was applied to one of the pieces of equipment (production unit) under study. This unit (Ref. *B802 pump motor*) comprises an electric motor (25 kW, 400 V, 2965 rpm) and a centrifugal pump and was selected because the large amount of data available makes the statistical modelling quite reliable.

After properly classifying the vibration measurement data, two relative frequency histograms were created, one for the motor and the other for the pump, to obtain the representations shown in Figs. 12 and 13.

Following this, a thorough analysis of the vibration measurements and the historical record of faults detected in the equipment was conducted to determine the sets of measurements necessary to obtain the real load *f(x)* and resistance *g(x)* distributions. This allowed the minimum value for which a defect was detected in the machines to be determined, as shown in Figs. 14 and 15, that is the real location of the corresponding $l_{p,\min}$* levels.

The next step is to model the real data according to Rayleigh dis-

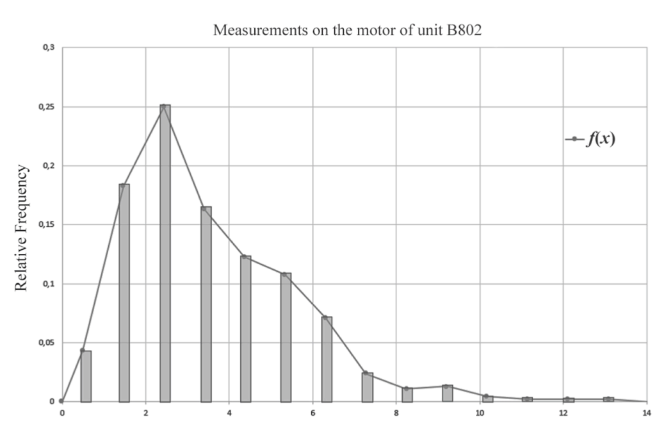

Fig. 12. Histogram of vibration level measurements of the motor of unit B802.

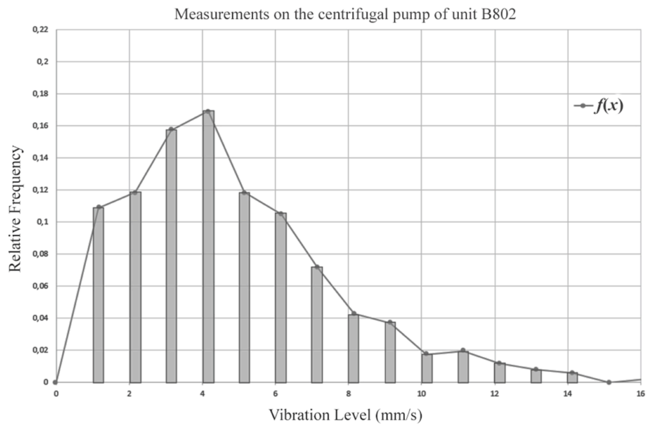

Fig. 13. Histogram of vibration level measurements of the centrifugal pump of unit B802.

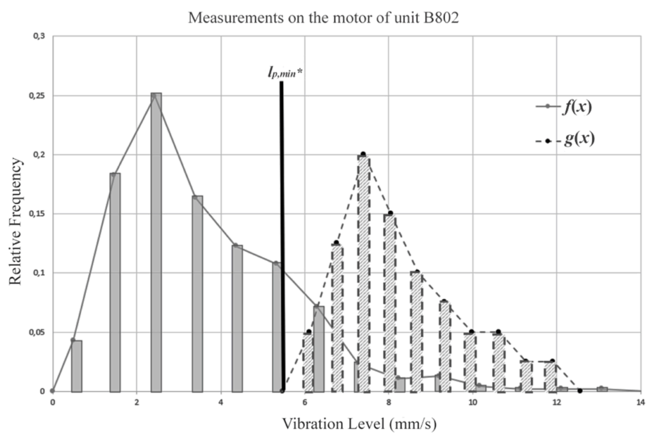

Fig. 14. Overlap of the load and resistance histograms to locate the corrected minimum limit level $l_{p,\min}$* (alert level) of the motor of unit B802.

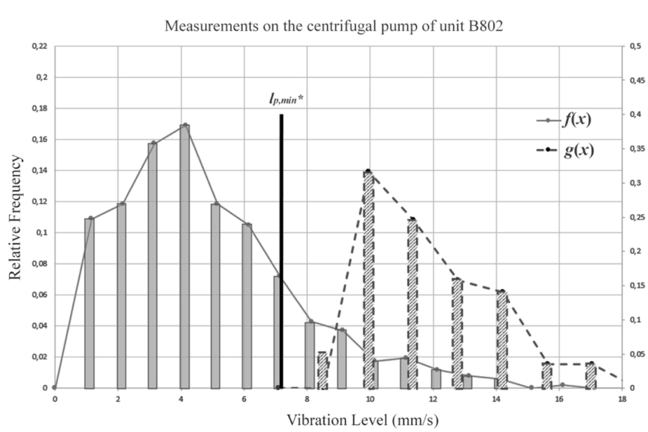

Fig. 15. Overlap of the load and resistance histograms to locate the corrected minimum limit level $l_{p,\min}$* (alert level) of the centrifugal pump of unit B802.

tribution. To do this, once the arithmetic mean of the real data ($\overline{x}$) has been obtained, assuming that it constitutes a correct estimate of the theoretical mean (μ) of the equipment in question, the mode (m) and standard deviation (σ) of the distribution are calculated using Eqs. (14) and (15). The results are indicated in Table 2.

Once the value of the mode is calculated, the theoretical model of each machine in the category can be obtained and represented, as shown in Figs. 16 and 17.

Table 2
Estimated statistical properties of the model.

	Motor (B802)	*Pump (B802)*
Mean (μ)	*3.24*	*3.99*
Mode (m)	*2.59*	*3.19*
Standard deviation (σ)	*1.695*	*2.09*

5.2. Statistical validation of the Rayleigh model applied to the case

As was done in the general description of the method (Section 3), and to explain in detail the arguments that support them and their effectiveness, the complete process of verification of the fit is developed for this real example case, with the corresponding goodness-of-fit tests being to the *Kolmogorov-Smirnov Test*. The level of significance chosen was the classic one for this type of analysis, that is, a value of 0.05, thus obtaining critical values for the motor and pump of 0.054 and 0.067, respectively. The following Tables 3 and 4 present the results of both analyses. In each case, the difference between the values of the distribution functions $F(x)$ and $F(x)_{Theoretic}$, as well as the corresponding value of the parameter KS_{value} against which the critical values are compared.

As can be seen, in both cases, the necessary condition for accepting the initial hypothesis is met, as for the motor $KS_{value} = 0.046 < V_{cr}\mathrm{i}_{tical} = 0.054$, and for the pump $KS_{value} = 0.056 < V_{critical} = 0.067$, and, therefore, it can be stated that the Rayleigh function adequately models the distribution of real data for both machines.

5.3. Modelling the case

After this verification, two representations were made in which the theoretical distribution functions of each machine were included, together with some of their most significant statistical properties and previously determined $l_{p,\min}$* levels, as shown in Figs. 18 and 19.

In these figures it can be seen that the vibration level $l_{p,\min}$* takes values of 5.75 mm/s and 7.21 mm/s for the motor and the centrifugal pump respectively. In this way, as indicated in Section 4.5, the corrected limit levels $l_{p,1}$*, $l_{p,2}$* y $l_{p,3}$*, corresponding to the alert, alarm, and stop levels, respectively, for the motor and the pump of the example, are those indicated in Table 5. This table indicates the values obtained when performing complete statistical modelling, that is, what has been described in this section ($l_{p,\min}$*, $l_{p,2}$*, $l_{p,3}$*), together with the values calculated by directly applying the method ($m + 2\sigma$, $m + 4\sigma$, $m + 8\sigma$), as explained in Section 4.6.

As can be seen in Table 5, and being a conclusion of great relevance in this work, it is necessary to highlight the fact that the level $l_{p,\min}$*

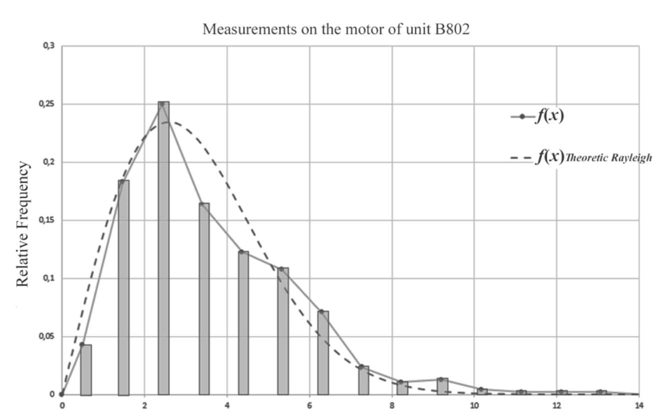

Fig. 16. Histogram of real data and its corresponding Rayleigh distribution model, $f(x)_{Theoretic}$, obtained for the motor of unit B802.

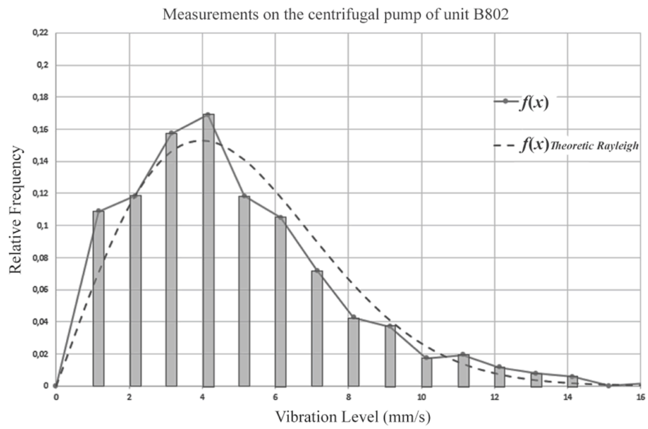

Fig. 17. Histogram of real data and its corresponding Rayleigh distribution model, $f(x)_{Theoretic}$, obtained for the centrifugal pump of unit B802.

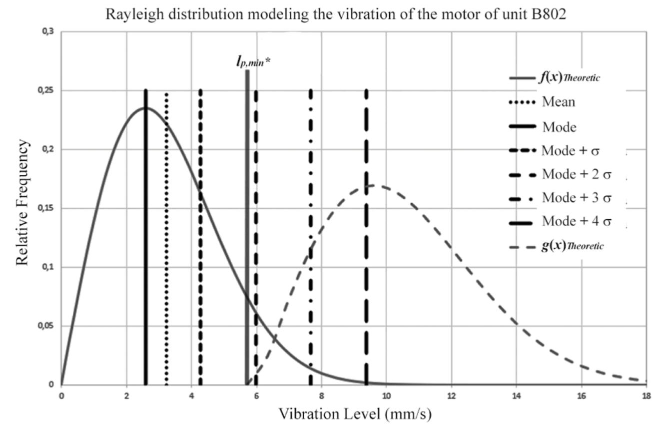

Fig. 18. Overlap of load and resistance theoretic distributions obtained for the motor of unit B802, showing the location of the alert level (*lp,*min*) and other significant statistical properties.

Table 3
Determination of the KS_{value} value for the *B802* pump motor.

$F(x_i)$	$F(x_i)_{Theoretic}$	$\lvert F(x_i) - F(x_i)_{Theoretic}\rvert$
0.000	0.000	0.000
0.041	0.059	0.018
0.211	0.213	0.002
0.461	0.415	**0.046**
0.645	0.614	0.031
0.785	0.774	0.011
0.892	0.882	0.010
0.953	0.945	0.007
0.970	0.978	0.008
0.983	0.992	0.009
0.989	0.997	0.008
0.991	0.999	0.008
0.994	1.000	0.006
0.996	1.000	0.004
0.998	1.000	0.002
0.998	1.000	0.002
0.998	1.000	0.002
1.000	1.000	0.000
$KS_{VALUE} = 0.046$		

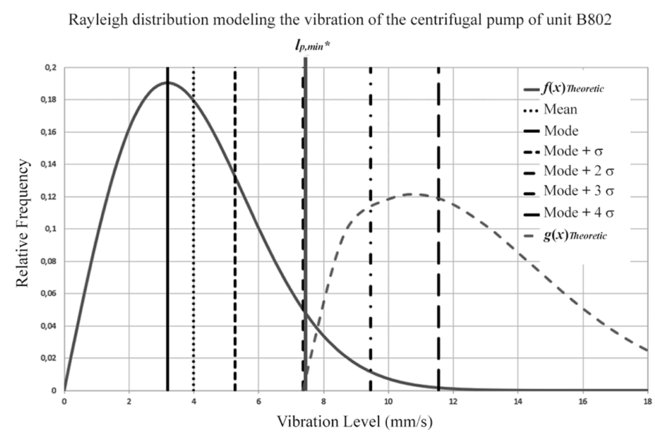

Fig. 19. Overlap of load and resistance theoretic distributions obtained for the centrifugal pump of unit B802, showing the location of the alert level (*lp,*min*) and other significant statistical properties.

Table 4
Determination of the KS_{value} value for the *B802* centrifugal pump.

$F(x_i)$	$F(x_i)_{Theoretic}$	$\lvert F(x_i) - F(x_i)_{Theoretic}\rvert$
0.000	0.000	0.000
0.109	0.071	0.037
0.227	0.191	0.036
0.384	0.339	0.046
0.549	0.493	**0.056**
0.672	0.635	0.037
0.777	0.754	0.023
0.849	0.844	0.004
0.891	0.907	0.016
0.928	0.948	0.020
0.946	0.973	0.027
0.965	0.987	0.022
0.979	0.994	0.015
0.988	0.997	0.009
0.995	0.999	0.004
1.000	1.000	0.000
$KS_{value} = 0.056$		

Table 5
Mean, mode and standard deviation values of the motor and pump of unit *B802*.

	Motor	Pump
$l_{p,\min}^*$ / $m + 2\sigma$	5.75 / 5.98	7.21 / 7.57
$l_{p,2}^*$ / $m + 4\sigma$	9.33 / 9.37	11.55 / 11.56
$l_{p,3}^*$ / $m + 8\sigma$	16.09 / 16.15	19.01 / 19.91

obtained from the real data has a value sufficiently close to the proposed value for establishment as the first limit level, which is the value $m + 2\sigma$ calculated from the statistical properties of the model function $f(x)_{Theoretic}$.

Since the B802 is a 25 kW unit, according to the general ISO 20,816–1 standard, it should have an initial alert value of 2.8 mm/s and alarm level of 7.1 mm/s. However, as displays Table 5, the alert and alarm levels calculated (the first two) for each of the machines in this unit are clearly higher than the standard, allowing for much better and more realistic monitoring of this unit.

As seen, the described methodology follows a fairly simple systematic process, as it can be applied as soon as a sufficient set of measurements is available, it should be noted that not only the machine limits can be calculated, as shown in Table 5, but also specific vibration limit levels can be defined for each specific point of the machine, which contributes to significantly improving the diagnostic accuracy.

As shown in Table 6, where the letter B in the reference code stands for pump, BV for vacuum pump and M for electric motor, the average difference between one value and another is less than 6 % for the group of machines in the industrial plant analysed in this example, which not only confirms that the choice of this first level is correct but also supports previous studies [44] that pointed to this hypothesis directly but lacked

Table 6
Comparison between the values of real and theoretical levels of fault detection.

Machine	*Mean*	*Mode*	*Real* $l_{p,\min}$*	$m + 2\sigma$
B801	4.21	3.35	8.02	7.74
B802	3.99	3.19	7.21	7.37
B803	4.72	3.77	9.25	8.71
B804	5.19	4.14	9.05	9.56
B805	5.55	4.43	9.57	10.23
B806	5.55	4.43	9.31	10.23
B807	1.67	1.33	3.12	3.07
B808	3.39	2.71	6.00	6.26
B809	3.99	3.20	6.87	7.39
B810	3.18	2.54	5.94	5.87
B811	2.36	1.88	4.00	4.34
B812	3.59	2.86	7.51	6.61
BV101	2.22	1.78	3.48	4.11
BV102	1.57	1.25	3.58	2.89
BV103	1.76	1.41	3.36	3.26
BV104	1.91	1.52	3.36	3.51
BV105	1.35	1.08	2.96	2.50
BV106	1.76	1.41	3.60	3.26
BV107	2.92	2.34	5.59	5.41
BV108	2.30	1.84	4.47	4.25
BV109	3.02	2.41	6.00	5.57
BV110	2.55	2.04	4.56	4.71
BV111	2.24	1.79	4.54	4.14
BV112	2.75	2.20	4.96	5.08
M801	5.61	4.47	9.77	10.33
M802	3.24	2.59	5.75	5.98
M803	3.57	2.85	6.87	6.58
M804	5.10	4.05	8.75	9.36
M805	3.76	3.00	7.22	6.93
M806	3.16	2.52	5.46	5.82
M807	2.82	2.25	5.53	5.20
M808	3.22	2.56	6.43	5.91
M809	3.10	2.47	6.05	5.71
M810	3.33	2.67	6.27	6.17
M811	3.25	2.58	5.60	5.96
M812	4.46	3.56	8.30	8.22
M101	1.36	1.08	2.60	2.50
M102	2.58	2.06	4.80	4.76
M103	3.00	2.39	5.59	5.52
M104	2.47	1.97	4.25	4.55
M105	1.95	1.56	4.17	3.60
M106	3.49	2.78	6.40	6.42
M107	1.93	1.54	3.75	3.56
M108	2.65	2.12	4.83	4.90
M109	2.96	2.36	5.56	5.45
M110	2.74	2.19	4.96	5.06
M111	2.36	1.88	4.56	4.34
M112	2.57	2.05	4.60	4.74

mathematical support to justify it. As in the example in Table 5, for the pump and motor of unit B802, the values of $m + 2\sigma$ in Table 6, which represent the first alert level, are specific for each machine and, in some cases, quite different from those established by the standard for machines of that type and power.

Finally, Table 7 shows a comparison between the alert and alarm limits, that is, the vibration boundaries that define Zones C and D, respectively, given by the ISO 20,816 standard (for each machine according to its type) and those obtained using the proposed statistical model. According to that standard, a vibration level in Zone C is considered unsatisfactory for long-term continuous operation, as a first alert level, while vibration values in Zone D, or alarm level, could cause damage to the machine. The new thresholds calculated using the proposed method are $m + 2\sigma$ and $m + 4\sigma$, respectively, in the model adapted to each machine. The significant differences between the two sets of values underscore the importance of adapting the levels to each particular machine to avoid continuous false alarms during monitoring, which cause inconvenience, and even interruptions, in production.

As can be seen in Table 7, all machine alert levels in the corrected model are higher than those in the ISO standard. However, six alarm levels with the new boundaries have lower values than their

Table 7
Comparison between the alert and alarm limits given by the ISO 20,816 standard (for each machine according to its type) and those obtained through their statistical model.

Machine	*ISO Alert*	*ISO Alarm*	m	σ	$m + 2\sigma$	$m + 4\sigma$
B801	**2.8**	**7.1**	3.35	2.19	**7.74**	**12.13**
B802	**2.8**	**7.1**	3.19	2.09	**7.37**	**11.55**
B803	**2.8**	**7.1**	3.77	2.47	**8.71**	**13.65**
B804	**4.5**	**11.2**	4.14	2.71	**9.56**	**14.98**
B805	**4.5**	**11.2**	4.43	2.90	**10.23**	**16.03**
B806	**4.5**	**11.2**	4.43	2.90	**10.23**	**16.03**
B807	**1.8**	**4.5**	1.33	0.87	**3.07**	**4.81**
B808	**2.8**	**7.1**	2.71	1.78	**6.26**	**9.81**
B809	**2.8**	**7.1**	3.20	2.10	**7.39**	**11.58**
B810	**2.8**	**7.1**	2.54	1.66	**5.87**	**9.20**
B811	**1.8**	**4.5**	1.88	1.23	**4.34**	**6.80**
B812	**2.8**	**7.1**	2.86	1.87	**6.61**	**10.36**
BV101	**1.8**	**4.5**	1.78	1.17	**4.11**	**6.44**
BV102	**1.8**	**4.5**	1.25	0.82	**2.89**	**4.53**
BV103	**1.8**	**4.5**	1.41	0.92	**3.26**	**5.11**
BV104	**1.8**	**4.5**	1.52	1.00	**3.51**	**5.50**
BV105	**1.8**	**4.5**	1.08	0.71	**2.50**	**3.92**
BV106	**1.8**	**4.5**	1.41	0.92	**3.26**	**5.11**
BV107	**1.8**	**4.5**	2.34	1.53	**5.41**	**8.48**
BV108	**1.8**	**4.5**	1.84	1.21	**4.25**	**6.66**
BV109	**2.8**	**7.1**	2.41	1.58	**5.57**	**8.73**
BV110	**2.8**	**7.1**	2.04	1.34	**4.71**	**7.38**
BV111	**2.8**	**7.1**	1.79	1.17	**4.14**	**6.49**
BV112	**2.8**	**7.1**	2.20	1.44	**5.08**	**7.96**
M801	**2.8**	**7.1**	4.47	2.93	**10.33**	**16.19**
M802	**2.8**	**7.1**	2.59	1.70	**5.98**	**9.37**
M803	**2.8**	**7.1**	2.85	1.87	**6.58**	**10.31**
M804	**4.5**	**11.2**	4.05	2.65	**9.36**	**14.67**
M805	**4.5**	**11.2**	3.00	1.97	**6.93**	**10.86**
M806	**4.5**	**11.2**	2.52	1.65	**5.82**	**9.12**
M807	**1.8**	**4.5**	2.25	1.47	**5.20**	**8.15**
M808	**2.8**	**7.1**	2.56	1.68	**5.91**	**9.26**
M809	**2.8**	**7.1**	2.47	1.62	**5.71**	**8.95**
M810	**2.8**	**7.1**	2.67	1.75	**6.17**	**9.67**
M811	**1.8**	**4.5**	2.58	1.69	**5.96**	**9.34**
M812	**2.8**	**7.1**	3.56	2.33	**8.22**	**12.88**
M101	**1.8**	**4.5**	1.08	0.71	**2.50**	**3.92**
M102	**1.8**	**4.5**	2.06	1.35	**4.76**	**7.46**
M103	**1.8**	**4.5**	2.39	1.57	**5.52**	**8.65**
M104	**1.8**	**4.5**	1.97	1.29	**4.55**	**7.13**
M105	**1.8**	**4.5**	1.56	1.02	**3.60**	**5.64**
M106	**1.8**	**4.5**	2.78	1.82	**6.42**	**10.06**
M107	**1.8**	**4.5**	1.54	1.01	**3.56**	**5.58**
M108	**1.8**	**4.5**	2.12	1.39	**4.90**	**7.68**
M109	**2.8**	**7.1**	2.36	1.55	**5.45**	**8.54**
M110	**2.8**	**7.1**	2.19	1.43	**5.06**	**7.93**
M111	**2.8**	**7.1**	1.88	1.23	**4.34**	**6.80**
M112	**2.8**	**7.1**	2.05	1.34	**4.74**	**7.43**

corresponding values in the ISO standard (machines highlighted in bold). In practice, this means that these six machines would warn of an alarm condition earlier than the ISO standard, while in the other cases, the number of false alarms or alerts (false negatives) will be considerably lower.

6. Conclusions

Throughout this work, it was demonstrated that the measurement of the vibration level in a rotating machine can be modelled statistically with a high degree of fit using the Rayleigh distribution function. The mode of said distribution function is the basic statistical property to characterise the behaviour of the machine and, consequently, the key to establish successive threshold levels between functional categories, such as the alert, alarm, and stop levels.

Although some standards, notably the ISO 20,816 vibration standard, give a widely validated initial approach for assessing the functional conditions of machines, as the standard itself recommends, when sufficient and reliable data are available to characterise each machine, it is preferable to "adjust the limit levels" to the particularities of each machine. However, there is no proposed method for doing so in either this standard or the related literature, which is novelty of this study.

The methodology presented in this work provides a tool that, in a systematic manner, allows the characterisation of the vibration measurements, taken with accelerometers, of each machine and establishes different threshold reference levels, that is, their own indicators for warning regarding the machine's condition. From a practical standpoint, this represents a significant reduction in false-positive alerts and, more importantly, a greater accuracy in fault detection, that is, a decrease in false negatives, since each machine will use its own reference level, based on its historical data, to evaluate its measurements.

After analysing hundreds of dynamic rotating equipment over more than twelve years, with thousands of vibration measurements for each of them, as necessary support for a reliable statistical approach, this work proposes a mathematical formulation that constitutes a generalisation of the proposals already suggested by previous studies, specifically to establish the successive limit levels for machine malfunction through vibration measurements.

In short, the main findings of this study and their importance can be summarized as follows:

- The Rayleigh distribution function has proven to be the best statistical model of the vibration level in a rotating machine.
- The mode, m, of said statistical distribution function constitutes de basal vibration level to characterise its functional condition.
- The mode, m, of the Rayleigh model can be calculated from the discrete average of a set of reliable measurements of the machine's vibration level, which is very easy in practice.
- Based on the Load-Resistance analysis, the different categories of functional status of a machine (good, alert, alarm…) have been defined using the expression $m + 2n\sigma$, where m and σ are, respectively, the mode and standard deviation of the Rayleigh distribution, and n is the order of the boundary level to be set. Therefore, $m + 2\sigma$ constitutes the first vibration alert value of the machine.
- Above the second-order limit level, $m + 4\sigma$, machine vibration can be considered an outlier, meaning a problem or abnormal functional condition in the machine, with an associated probability of 99.8 %, a value traditionally considered a threshold in many other technical fields.
- This methodology has been tested with great success on hundreds of rotating machines in different industrial plants, demonstrating that is very useful, quick and easy to implement in the predictive maintenance programs.

Funding

This work has been partially supported by the Research Project 34,126 "*INTELLIGENT DIAGNOSIS OF DYNAMIC EQUIPMENT IN INDUSTRY 4.0 (iDIANA4.0–2020)*" (art. 83 L.O.E.), Ref. JUSTO-34,126 of the University of Murcia (Spain).

CRediT authorship contribution statement

J Sánchez Robles: Writing – original draft, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **FC Gómez de León Hijes:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **AR Pastor Más:** Supervision, Investigation, Data curation. **FM Martínez García:** Supervision, Project administration, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that support the findings of this study are available from the corresponding author upon request..

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